UDC 533;538

MAGNETIC SUSPENSIONS AS MICROPOLAR FLUIDS

PMM Vol. 43, No. 3, 1979, pp. 574-576

V. M. SUIAZOV
(Voronezh)

(Received May 7, 1976)

Polarizable and magnetizable continuous media (elastic solids, gases, or fluids) with an intrinsic angular momentum have been attracting the attention of researchers for some considerable time. One of the earliest publication in which such media were considered without allowance for electromagnetic effects is the monograph [1]. Numerous further works had appeared since 1943 in which theories of continuous media with intrinsic angular momentum were developed without taking into account electromagnetic effects. Below we shall refer only to publications in which allowance is made for electromagnetic effects (*). Equations defining deformable polarizable and magnetizable media were derived in [2] in the case when the intrinsic angular momentum and magnetization intensity connected by a finite relation. However the results of that work are not considered here. The general theory of construction of models of deformable polarizable and magnetizable media based on thermodynamics was presented in [3, 4].

The hydrodynamics of magnetic suspensions in the form of ferromagnetic particles in a fluid carrier is one of the domains of continuous medium mechanics in which it is important to take into consideration the inner mechanical moment and magnetization. Magnetic suspensions are, moreover, examples of media for which there may be no finite relation between the magnetization intensity vector $\mathbf{M}$ and vector $\mathbf{w}$ of the particle angular velocity of rotation. These aspects of the problem are taken into account in the theory developed by the author in [5]. A system of equations for defining the motions of a magnetizable fluid with intrinsic angular momentum was presented by Shliomis and Zaitsev in [6] without proof. In that paper the equation for magnetization intensity is given as

$$\frac{\partial \mathbf{M}}{\partial t} = -\frac{1}{\tau}\left(\mathbf{M} - M_0 \frac{\mathbf{H}}{H}\right) \tag{1}$$

Such equation does not correctly define the phenomena occurring during the motion of a deformable medium, if the intrinsic angular momentum is taken into account.

In a paper published by Shliomis in December 1971 [7] the partial derivative in Eq. (1) was replaced by the total derivative and the term $\mathbf{w} \times \mathbf{M}$ was added to it without explanation or reference. That term had been already derived by the author

*) There are no suitable mechanisms in the theories of gas, fluid, or solids for dealing with the essence of electromagnetic effects in this connection.

from thermodynamic principles, and published in April 1970 [5] (*).

The system of equations that appeared in [5] in the particular case of incompressible magnetizable but nonpolarizable (when the transport coefficients can be assumed constant) micropolar fluid in the absence of distributed charges is

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + (\alpha_2 + \alpha_3)\nabla\cdot\nabla\mathbf{v} + 2\alpha_3\nabla\times\mathbf{w} -$$
$$\alpha_4\mu_0\nabla\left(\frac{d\mathbf{m}}{dt} - \mathbf{w}\times\mathbf{m}\right) + (\mu_0\mathbf{m}\cdot\nabla)\,\mathbf{h} - \mu_0\frac{d\mathbf{m}}{dt}\times\varepsilon_0\mathbf{e} - \mathbf{v}\times(\mu_0\mathbf{m}\cdot\nabla)\,\varepsilon_0\mathbf{e} + \rho\mathbf{f}$$
$$\rho J\frac{d\mathbf{w}}{dt} = 2\alpha_3(\nabla\times\mathbf{v} - 2\mathbf{w}) + 2\alpha_4\mu_0\left(\frac{d\mathbf{m}}{dt} - \mathbf{w}\times\mathbf{m}\right) + \tag{2}$$
$$\mathbf{m}\times\mathbf{h} + (\gamma_1 - \gamma_3 + \gamma_2/3)\,\nabla\nabla\cdot\mathbf{w} + (\gamma_3 + \gamma_2)\,\nabla\cdot\nabla\mathbf{w}$$
$$\frac{d\mathbf{m}}{dt} = \mathbf{w}\times\mathbf{m} + \frac{1}{\tau}(\mathbf{h}K - \mathbf{m}) + \alpha_4^\circ(\nabla\times\mathbf{v} - 2\mathbf{w})$$

where $\tau = \mu_0 h_1 K$, $\alpha_4^\circ = \alpha_4 / \mu_0 h_1$ are the relaxation time and the coefficient of gyromagnetic coupling, h_1 is a constant, $\mathbf{f}$ is the external mass force density, $\mathbf{v}$ is the velocity of motion, α_2 and α_3 are the shear and rotation viscosities, $\mathbf{e}$ and $\mathbf{h}$ are vectors of electric and magnetic field intensities, J is the moment of inertia of particles, ∇ is a Hamiltonian operator, and ε_0 and μ_0 are the electric and magnetic space constants. Equations (2) must be supplemented by equations of magneto-electrostatics.

Using the notation

$$\mathbf{m} = \mathbf{M},\ \mathbf{h} = \mathbf{H},\ \mathbf{w} = \frac{\mathbf{S}}{I},\quad J = I,\ \alpha_2 = \eta,\ \alpha_3 = \frac{\rho I}{4\tau_S},\quad \tau = \tau_B,\ K = \frac{M_0}{H} \tag{3}$$

and the supplementary assumptions about the absence of electric field, of the gyromagnetic cross effect, and of the flux of the intrinsic angular momentum, i.e. setting in Eqs. (2)

$$\mathbf{e} = 0,\quad \alpha_4 = 0,\quad \gamma_1 = \gamma_2 = \gamma_3 = 0 \tag{4}$$

we obtain a system of equations that exactly coincide with the equations appearing in Shliomis's paper [7], as well as with Eqs. (17)–(19) in [8] of which Shliomis is one of the authros. It was stated in that paper that "these equations ((17)–(19) of Shliomis) differ from those obtained in [75–77]," i.e. from those in the author's papers [5, 9, 10]. This implies that "these" equations differ from the equations proposed by the author in [5] only in that they are a particular case of these and are directly derived from them by the introduction of the additional constraints (4).

In fact, these additional assumptions are not always admissible (for instance, it is not possible to define the effect of drag exerted on a ferrosuspension by a rotating homogeneous magnetic field under conditions (4)). No proof of their validity in any specific cases, including those considered by Shliomis, is given.

The statement that "...phenomenological definition of magnetic suspensions cannot be generally effected in the case of arbitrary external effects, and properties and

*) In 1969 the author submitted to the PMM Editorial Board papers on the hydrodynamics of nonisothermal charged, conducting, and polarizable media, in which allowance was made for the inhomogeneity of magnetization and polarization, and for microdeformations and their rate. These papers were strongly criticized by Shliomis, and were subsequently rejected by the Editorial Board.

degrees of dispersion of the ferromagnetic" which appeared in Shliomis's paper [11] does not agree with facts. The equations for such media that appeared in [3–5, 9, 10] were derived on phenomenological grounds and on the basis of general theories. The writing of these equations "by intuition" without analysis, as was done, for instance, in [6], leads to errors that had to be later rectified in conformity with the general theory [5] which also implies the necessity of taking into consideration other important terms.

REFERENCES

1. Cosserat, E. and Cosserat, F., Théorie des Corps Déformables. Paris, Hermann et Fils, 1909.

2. Vlasov, K. B. and Ishmukhametov, B. Kh., The equations of motion and state of magnetoelastic media. ZhETF, Vol.46, No.1, 1964.

3. Sedov, L. I., On ponderomotive forces of interaction of an electromagnetic field and accelerating material continuum, taking into account finite deformations. PMM, Vol.29, No.1, 1965.

4. Sedov, L. I., Some problems of designing new models of continuous media. Proc. 11th Internat. Congr. Appl. Mech., 1964. Berlin, Springer, 1966.

5. Suiazov, V. M., On the asymmetric model of a viscous electromagnetic fluid. PMTF, No.2, 1970.

6. Zaitsev, V. M. and Shliomis, M. I., The dragging of ferromagnetic suspension by a rotating field. PMTF, No.5, 1969.

7. Shliomis, M. I., Effective viscosity of magnetic suspensions. ZhETF Vol. 61, No.6, 1971.

8. Shaposhnikov, I. G. and Shliomis, M. I., Hydrodynamics of magnetizable media. Magnetic Hydrodynamics, No.1, 1975.

9. Suiazov, V. M., On the equtions of motion of interpenetrating electromagnetic fluids. Magnetic Hydrodynamics, No.2, 1971.

10. Suiazov, V. M., On the structurally continuous approach to the magneto- and electro-rheology of disperse fluid systems. Magnetic Hydrodynamics, No. 2, 1972.

11. Shliomis, M. I., Magnetic fluids. Uspekhi Fiz. Nauk, Vol.112, No.3, 1974.

Translated by J. J. D.